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To Determination of Stresses in the Stretched Armature of Extracredly Compressed Elements in the Limit Condition

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Abstract. The problem of determining stresses in a stretched reinforcement of inflexible eccentric-compressed reinforced concrete elements is considered when the height of the compressed zone in the limit state exceeds its boundary value corresponding to the condition of equal strength of the section. Instead of the generally accepted linear dependence of the stresses under consideration on the height of the compressed zone, an elliptic relationship more expedient from different points of view has been proposed.

Introduction

Non-centrally compressed reinforced concrete elements with the resultant external forces, passing outside the core section and causing the formation of different-valued stress patterns, form a fairly wide class of structures. When calculating the strength of the stress elements in the tensioned reinforcement in the limit state are determined depending on the degree of reinforcement of the cross section and the ratio of the calculated height of the compressed zone and its boundary value. In the case when the calculated relative height of the compressed zone ξ in the limiting state is less than the boundary relative height of the compressed zone ξR, which corresponds to the simultaneous transition to the limiting state of both compressed concrete and stretched reinforcement, the stress in the stretched reinforcement σs reaches the limiting value R_s. Otherwise, the destruction of the structure occurs on compressed concrete with the underutilized strength of the tensioned reinforcement. This case is realized under conditions of non-central compression with a small eccentricity, when the resultant external load is within the core of the section, or under conditions of section saturation with an excess amount of tensioned reinforcement.

Used in modern methods ([1] - [3]) for practical calculations, the calculation of stresses σs in a stretched reinforcement in the limit state at $\xi > \xi R$ using a linear formula between the extreme values of stresses from $+Rs \cdot (at \cdot \xi \le \xi R)$ to $-Rs$ (at fully compressed section, $\xi \geq 1$) leads to underestimated values of stresses in the reinforcement and even a change of sign in conditions when physically the stress in the reinforcement located in the tensioned zone cannot be compressive. In addition, since at the point $\xi = \xi R$ on the abscissa of the $\sigma s / Rs = f(\xi)$ graph there are no physical factors that could cause a break or rupture of the function, the conjugation of two sections of this function to the left and right of the point $\xi = \xi R$ should be smooth which causes the convexity of the curve $\sigma s / Rs = f(\xi)$ on the segment $\xi R \leq \xi \leq 1$.

In this paper, the disadvantages generated by the linearity of the function $\sigma s / Rs = f(\xi)$ are mitigated by using a nonlinear relationship between the stresses in the tensioned reinforcement and the height of the compressed zone in the limit state of an eccentric loaded reinforced concrete element.

Purpose of the Study

The large error in the determination of stresses in the reinforcement in the limiting state with a linear dependence $\sigma s / Rs = f(\xi)$ causes an increase in the consumption of reinforcement, which is important from a practical point of view.

On the other hand, calculations show that, for example, at $\xi R = xR / h0 = 0.4$, where xR is the boundary height of the compressed zone, $h0$ is the working height equal to the height of the section h minus the thickness of the concrete protective layer a ($h0 = h - A$), the stresses in the reinforcement of the stretched zone in the limiting state according to the accepted linear formula ([1] - [3]) turn out to be close to zero already at the current value of the relative height of the compressed zone $\xi = 0.7$. Physically, this is not possible with such a relatively large stretch zone (about 30%). This theoretical inconsistency testifies to the inadmissibility of the linear approximation of the stresses in the reinforcement in the limit state as a function of the relative height of the compressed zone ξ.

The aim of the study is to replace the adopted linear dependence $\sigma s / Rs = f(\xi)$ with a non-linear, more appropriate to the character of the stress-strain state of the reinforced concrete structure at the marginal stage.

Material and Research Methods

The subject of research in this paper are the stresses in the stretched reinforcement of inflexible eccentrically compressed reinforced concrete elements in a limiting state with relatively large eccentricities forming a different diagram of stresses. Such conditions allow us to extend the results to bendable elements.

Concrete and reinforcement are considered as materials, the deformation of which is described by two ideally elastic-plastic diagrams.

The research method is analytical. The empirical parameter used in modern normative literature ([3]) is used as a criterion for the transition to the limit state — the boundary height of the compressed zone x_R and its relative value $\xi_R = x_R/h_0$.

The requirement of bulge of a curved line $\sigma_s/R_s = f(\xi)$ on a piece $\xi_R \leq \xi \leq 1$ and smoothness of its mating to a line $\sigma_s/R_s = 1$ on a lot $\xi \leq \xi_R$ is answered with an elliptic curved line in the first quadrant of co-ordinate system (ξ, σ_s/R_s). We will take it as a function approximating the change in stresses in the tensioned reinforcement σ_s/R_s depending on relative height of the compressed working area ξ in a limiting stage.

Results of Research and their Discussion

The principle of strength calculation of eccentrically compressed elements is most clearly seen in the example of elements of rectangular cross section. Figure 1 shows the design scheme for this case.

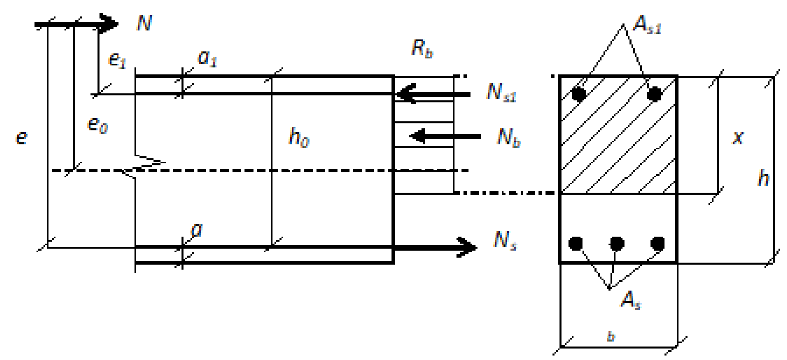

Fig. 1 The design scheme eccentrically compressed reinforced concrete element when calculating the strength

According to the calculation scheme of Figure 1, the equilibrium equations are as follows:

$$N+N_s-N_{s1}-N_b=0, \tag{1}$$
$$N_e-N_{s1}(h_0-a_1)-N_b(h_0-x/2)=0 \tag{2}$$

Where N is the resultant external load; $N_b=R_bbx=R_bbh_0\xi$ is the resultant of the forces in the concrete of the compressed zone, R_b - limiting resistance of concrete to compression, $\xi=x/h_0$ is the relative height of the compressed zone corresponding to the acting forces and properties of the materials of construction; $N_s=\sigma_sA_s$ =- force in tensioned reinforcement at the limit stage, σ_s - - calculated stress in tensioned reinforcement, which may be equal to or less than the limit value R_s, A_s, $\mu=A_s/(bh_0)$ – is a cross-sectional area and reinforcement ratio of tensioned reinforcement; $N_{s1}=R_{sc}A_{s1}=R_{sc}\mu_1bh_0$ is the force in compressed reinforcement, R_{sc} is the ultimate stress in compressed reinforcement associated with the effect of loss of form stability and depending on the deformation properties of concrete, A_{s1}, $\mu_1=A_{s1}/(bh_0)$ - is the cross-sectional area and the coefficient of reinforcement of compressed reinforcement; $x=\xi h_0$ is the height of the compressed zone; b, h, h_0 are the width, height and working height of the cross section, $h_0=h-a, a, a_1$ - is the thickness of the protective layer for stretched and compressed reinforcement, e, e_1 are the distance from the resulting external load to the axes of stretched and compressed reinforcement, respectively.

Usually, when solving a direct problem, the external load N, its eccentricities with respect to tensioned and reinforced e_1 reinforcement are known, cross-sectional sizes b, h, h_0 and cross-sectional area of compressed reinforcement A_{s1}, assigned for structural reasons, it is necessary to determine the required amount of stretched reinforcement A_s. In this case, in the equation (1), in the general case, there are three unknown parameters: the height of the compressed zone in the limiting state $x=\xi h_0$, the required amount of stretched reinforcement $A_s=\mu bh_0$ and pressure in it σ_s. In the particular case when the compressed zone height corresponding to the conditions of the problem in the limiting state x is not greater than the limiting value of the compressed zone height x_R (or in relative sizes $\xi\leq\xi_R$), the stress in the tensioned reinforcement is equal to the limiting $(\sigma_s=R_s)$, and number of unknown persons in the equation (1) is reduced to two. In the equation (2) which expresses the sum of the moments of all forces concerning a centroid of the stretched armature, one contains only unknown x, as effort in the stretched armature, including generally two unknown persons (σ_s, A_s), no as it takes place through моментную a point.

Thus, from the equation (2) it is defined x and further from (1) if $\xi\leq\xi_R$ and accordingly $\sigma_s/R_s=1$, we find A_s. If $\xi>\xi_R$, that, as follows from the experiments, σ_s/R_s <1, and the stress σ_s σs in the reinforcement in the limiting state of the structure is also unknown, which translates the problem into a class of statically indeterminate and forces to introduce into the picture the deformations and the relations between the deformations and stresses. In the limiting state, this picture and these relations are extremely diverse and depend on many random factors. The way out of this difficulty is found in a purely engineering way, namely by introducing an integral parameter established experimentally - the boundary height of the compressed zone x_R (relative value $\xi_R=x_R/h_0$), at which the cross section of the structure is equal in strength, both in the compressed and stretched zone At $\xi\leq\xi_R$ $\sigma_s/R_s=1$, that is the stress σ_s in the tensioned reinforcement in the limit state is known and equal to the limit value R_s, in an interval $\xi_R\leq\xi\leq 1$ it is usually determined by the linear dependence $\sigma_s/R_s=f(\xi)$. How this is done can be seen from the graph of the function $\eta=\sigma_s/R_s=f(\xi)$, shown in Figure 2.

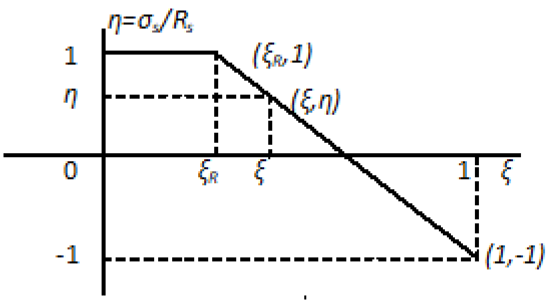

Fig. 2 Graph of the function of stresses in the tensioned reinforcement in the limit state depending on the relative height of the compressed zone

The line equation $\eta=\sigma_s/R_s=f(\xi)$ on a piece $\xi_R\leq\xi\leq 1$ has the form:

$$\frac{\xi-\xi_R}{1-\xi_R}=\frac{\eta-1}{-1-1}, \qquad \eta=1-2\frac{\xi-\xi_R}{1-\xi_R} \tag{3}$$

The expression (3) can be represented in two other forms:

$$\eta=1-\eta=1-2\frac{\xi-\xi_R}{1-\xi_R}=\frac{1-\xi_R-2\xi+2\xi_R}{1-\xi_R}=\frac{1+\xi_R-2\xi}{1-\xi_R} \tag{4}$$

$$\eta=\frac{1+\xi_R-2\xi}{1-\xi_R}=\frac{2-2\xi-1+\xi_R}{1-\xi_R}=2\frac{1-\xi}{1-\xi_R}-1. \tag{5}$$

Let us analyze the formula (8.13) "Set of rules. Concrete and reinforced concrete structures "[3], corresponding to the case $\xi > \xi_R$. In the notation adopted in this paper, this formula has the form:

$$x=\frac{N+R_sA_s\frac{1+\xi_R}{1-\xi_R}-R_{sc}A_{s1}}{R_bb+\frac{2R_sA_s}{h_0(1-\xi_R)}}. \tag{6}$$

Having performed the simple transformations of the formula (6), we get:

$$R_bbx+\frac{2R_sA_sx}{h_0(1-\xi_R)}=N+R_sA_s\frac{1+\xi_R}{1-\xi_R}-R_{sc}A_{s1}, \qquad R_bbx+\frac{2R_sA_s\xi}{1-\xi_R}=N+R_sA_s\frac{1+\xi_R}{1-\xi_R}-R_{sc}A_{s1},$$

$$N+R_sA_s\frac{1+\xi_R-2\xi}{1-\xi_R}-R_{sc}A_{s1}-R_bbx=0,$$

$$N+R_sA_s\frac{1+\xi_R-2\xi}{1-\xi_R}-N_{s1}-N_b=0. \tag{7}$$

Comparison of expressions (7) and (1) allows to write down

$$N_s=\sigma_sA_s=R_sA_s\frac{1+\xi_R-2\xi}{1-\xi_R}, \quad \sigma_s=R_s\frac{1+\xi_R-2\xi}{1-\xi_R}, \quad \eta=\sigma_s/R_s=\frac{1+\xi_R-2\xi}{1-\xi_R}$$

Note that the obtained expression for relative stresses in tensile $\eta=\sigma_s/R_s$ completely coincides with expression (4) and equivalent expressions (3), (5). Therefore, formula (7.13) from [3] is obtained according to the scheme of Figure 2.

Assuming the values $N, e, N_{s1}, h_0, a_1, b, h, R_b,$ given for one reason or another, we can from the equation (2) find the only unknown x.

To obtain and analyze the results, it is more convenient to present equations (1), (2), as in [4] - [6], in the dimensionless parameters obtained by dividing all the terms in equation (1) by R_bbh_0, and in the equation (2) - on $R_bbh_0^2$:

$$n_N+n_s-n_{s1}-n_b=0,$$
$$n_N+\sigma_s\mu/R_b-R_{sc}\mu_1/R_b-\xi=0, \tag{8}$$
$$n_N\zeta-n_{s1}(1-\alpha_1)-\xi(1-\xi/2)=0,$$
$$n_N\zeta-n_{s1}(1-\alpha_1)-\alpha_m=0, \tag{9}$$

where $n_N=N/(R_bbh_0)$, $n_s=N_s/(R_bbh_0)=\mu\sigma_s/R_b=(\mu\sigma_s/R_s)\ (R_s/R_b)=\mu\eta R_s/R_b$, $n_{s1}=N_{s1}/(R_bbh_0)=\mu_1R_{sc}/R_b$, $n_b=N_b/(R_bbh_0)=\xi$, $\zeta=e/h_0$, $\alpha_1=a_1/h_0$, $\alpha_m=\xi(1-\xi/2)$.

The relative height of the compressed zone ξ, corresponding to a given load and other source data, cannot be determined from equation (8), where it enters explicitly in the first degree, since this includes two other (generally unknown) values σ_s and μ. The relative height of the compressed zone ξ, corresponding to a given load and other source data, cannot be determined from equation (8), where it enters explicitly in the first degree, since this includes two other (generally unknown) values:

$$\alpha_m=n_N\zeta-n_{s1}(1-\alpha_1)=\xi(1-\xi/2), \tag{10}$$

and further

$$\xi=1-\sqrt{1-2\alpha_m} \tag{11}$$

The second root of the quadratic equation (10) with respect to ξ is discarded as not conforming to the condition of the distribution of stresses in the cross section.

We show that the radical expression in (11) is always positive.
Indeed,

$$d\alpha_m/d\xi=1-\xi,\ \ d^2\alpha_m/d\xi^2=-1,$$

means, at the extreme point $d\alpha_m/d\xi=0$, $\xi_{m,\ extr}=1$, $\alpha_{m,\ extr}=1*(1-1/2)=1/2$ and $\alpha_{m,\ extr}=1/2=max$ since at the extremum point the second derivative is negative.

In case:

$$\alpha_m=n_N\zeta-n_{s1}\left(1-\alpha_1\right)=\frac{Ne-R_{sc}A_{s1}(h_0-a_1)}{R_b b h_0^2}>0,5\,,$$

then this means that it is necessary to reduce the load N, eccentricity e, or increase the amount of compressed reinforcement A_{s1}, section dimensions h_0, b, concrete resistance to compression R_b.

Expanding in the Maclaurin series $\sqrt{1-2\alpha_m}$, converging at $\alpha_m\leq 0,5$:

$$\sqrt{1-2\alpha_m}\approx 1-\alpha_m-\alpha_m^{\ 2}/2-\alpha_m^{\ 3}/2-5\alpha_m^{\ 4}/2\ -.....,$$

it is possible to replace the formula (11) with an approximate expression

$$\xi=1-\sqrt{1-2\alpha_m}\approx\alpha_m+\alpha_m^{\ 2}/2+\alpha_m^{\ 3}/2+5\alpha_m^{\ 4}/2+....,\tag{12}$$

which is quite comprehensible at $\alpha_m\leq 0,4$, at $\alpha_m\leq 0,2$ allows to be limited to first two members, and at $\alpha_m\leq 0,1$ only the first member of decomposition (12).

From the equation (8) (or that is equivalent, from the equation (2)) we obtain the formula for calculation of demanded quantity of the stretched armature

$$\mu=(\xi+R_{sc}\mu_1/R_b,-n_N)R_b/\sigma_s. \tag{13}$$

Expression (13) shows that the required quantity μ of tensioned reinforcement is greater, the lower the stress in it in the limiting stage.

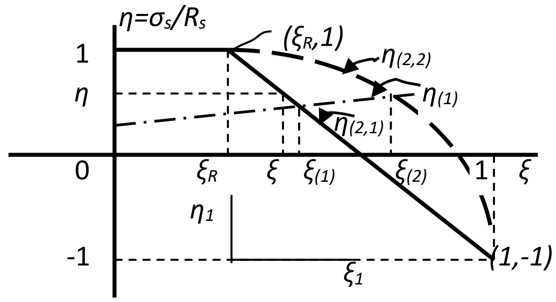

Fig. 3 Comparison of stresses in tensile reinforcement in the limit state with linear and non-linear description in the range of ξ from ξ_R to 1

In Figure 3, along with the straight line connecting the points *(ξ_R, 1)* and *(1,-1* of the graph of the function $\sigma_s/R_s=f(\xi)$ on the segment $\xi_R \leq\xi\leq 1$, an upward convexity curve is drawn for smooth conjugation with the graph of the function $\sigma_s/R_s=f(\xi$ in the previous segment $0\leq\xi\leq\xi_R$. . From this figure it is clearly seen that over the entire length of the segment $\xi_R \leq\xi\leq$ the curved line gives greater values of tensile stresses in the reinforcement than the straight line, and therefore, according to formula (13), less consumption of reinforcement.

As the simplest curve that meets the described general requirements, consider an ellipse. In a local coordinate system placed at the center of an ellipse, its equation will be:

$$\frac{\xi_1^2}{(1-\xi_R)^2}+\frac{\eta_1^2}{2^2}=1, \tag{14}$$

where ($1-\xi_R$) - the size of a horizontal semi-axis of the ellipse, and (1- (-1) =2) is the size of the vertical semi-axis according to Figure 3 In the general coordinate system (ξ, η) for the center of the ellipse we have:

$$\xi_1=\xi-\xi_R, \qquad \eta_1=\eta+1. \tag{15}$$

Accordingly, the equation of the ellipse in view of (15) will receive the form:

$$\frac{(\xi-\xi_R)^2}{(1-\xi_R)^2}+\frac{(\eta+1)^2}{2^2}=1. \tag{16}$$

After simple transformations from (16), we express the relative stress in the reinforcement η through the relative height of the compressed zone ξ in the limit state

$$\eta=2\sqrt{1-\left(\frac{\xi-\xi_R}{1-\xi_R}\right)^2}-1. \tag{17}$$

In the last expression of the plus and minus signs in front of the radical, only plus is left, since the equation of the ellipse in the first quadrant of the coordinate system (ξ_1, η_1) is considered.

Thus, the algorithm for solving the problem of determining the number of tensile reinforcement in an element that is eccentrically compressed with reinforced concrete with an unequal stress diagram is as follows:

1. from the moment equilibrium equation (9), the calculated relative height of the compressed zone ξ in the limiting state (formula (11)) is calculated;
2. If $\xi\leq\xi_R$, $\sigma_s/R_s=1$, if $\xi>\xi_R$, $\sigma_s/R_s=\eta$ (the formula (17));

3. The knowledge of the stresses in the tensioned reinforcement in the limit state now allows to find the required amount of this reinforcement from equation (8) of the balance of forces in the cross section (formula (13)).

Figure 3 gives a visual representation of the method of solving the problem in the proposed formulation and comparison of the results obtained with those given by the standard method. The dash-dotted line η (1) expresses the first equilibrium equation (8) written relative to the term $\sigma_s\mu/R_b$ in which σ_s it is replaced on ηR_s. The point of intersection of this line with the segment η (2,1), determines the values of ξ, η by the normative method. The point of intersection η (1) and the dotted arc of the ellipse η (2,2) determines the values of ξ, η by the proposed method. As can be seen from Figure 3, the difference in relative stress values in tensile reinforcement η is relatively small, but very significant in ξ values. Since the value of ξ is primary, since it was obtained directly from the equilibrium (moment) equation, then under the conditions that determine the position of the lines in the figure, the normative solution leads to a contradiction - to compressive stresses in the reinforcement of the stretched zone. The proposed method is free from this disadvantage.

The use of the nonlinear relationship (17) imposes the following constraints on the magnitude of the external load N ($n_N=N/(R_b b h_0)=\sigma_0/R_b$) and its eccentricity e ($\zeta=e/h_0$):

$$n_{s1}(1-\alpha_1)+\xi_R-0,5\,\xi_R^2<n_N\zeta<n_{s1}(1-\alpha_1)+0,491+0,018\,\xi_R-0,009\,\xi_R^2,- \quad (18)$$

which are derived in detail in [6]. Going beyond the lower boundary means going to the problem, where in the limiting state $\xi\leq\xi_R$, going beyond the upper boundary means that the stresses are all compressive.

The proposed nonlinear dependence of stresses in the tensioned reinforcement on the height of the compressed zone in the limit state of an eccentric squeezed reinforced concrete element (17) is more natural than the linear one (3), (5) from the physical point of view, since the limit equilibrium of the structure is considered. In addition, due to more reliable determination of stresses in tensile reinforcement, it may be useful to assess the limiting states of structures during repeated loads [7], elevated concrete classes [8,9], the study of long-term loads, when the initial state of the structure is close to the limiting [ten].

Summary

In work, the linear dependence of stresses in a tensile reinforcement of inflexible eccentrically compressed reinforced concrete elements as a function of the height of a squeezed zone in the limiting state, when it exceeds the boundary one, is replaced by a nonlinear elliptic dependence. The calculated formulas are obtained and the calculation algorithm is given. The proposed design scheme may be useful in solving various nonlinear problems in the theory of reinforced concrete with a high level of the initial stress-strain state, in particular, for problems with creep as an initial elastic-instantaneous solution or in problems of stability as a first approximation in an iterative process.

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