Cracking in Reinforced Concrete Structures of Buildings at Seismic Exposure

Mukhtar Bekkiev[1(✉)], Sergey Skuratov[2], Evgeniy Peresypkin[3], and Dmitry Vysokovsky[4]

[1] High-Mountain Geophysical Institute, pr. Lenina, 2, 360030 Nalchik, Russia
ps62@yandex.ru
[2] Sochi State University, Sovetskaya str., 26-A, 354000 Sochi, Russia
[3] Kuban State Agrarian University, Kalinin str., 13, 350004 Krasnodar, Russia
[4] Don State Technical University, pl. Gagarina, 1, 344002 Rostov-on-Don, Russia

Abstract. Considered numerical experiment model considering simplified-tion, and approximation on the basis of resonance-oscillatory scheme. The idea is based on the cross-shear diagrams of structural failure. Reduce their thickness or increase the possible load. The local nature of the action of the traveling wave leads to the fact that the main phenomena in the wave process are concentrated on the wave front, where the features in the distribution of deformations and forces are concentrated. When the transverse wave propagates, local shifts arise, generating transverse forces of large magnitude. The impact of a wave can significantly exceed the ability of a structure to absorb energy by forming fracture surfaces (cracks).

Keywords: Reinforced concrete · Building · Seismic exposure

1 Introduction

Often, the implementation of complex calculations of structures for seismic impacts is perceived as an enhanced guarantee of reliability. But the actual behavior of the structure in a strong earthquake depends on a large number of factors that are difficult to formalize in the form of mathematical models [1, 3–7]. The forecast of seismic loading characteristics, the reaction of the structure under alternating intensive dynamic impact are uncertain. Besides, there are no universal calculation models. In order to establish the limits of the applicability of a particular model, it is necessary to evaluate the accepted simplifications and approximations on the final result, as well as neglect of small quantities, which were considered as unimportant. For a complex system, it is rarely possible to establish an estimate of this kind, and not all hypotheses and simplifications have an experimental justification or are verifiable on a physical object, physical model. In this case, numerical experiments are necessary, in which certain parameters vary and the results obtained are estimated. Calculations are also needed for fundamentally different models, and not for different software complexes built on essentially one finite element model.

V. Murgul and Z. Popovic (eds.), *International Scientific Conference Energy Management of Municipal Transportation Facilities and Transport EMMFT 2017*, Advances in Intelligent Systems and Computing 692,
https://doi.org/10.1007/978-3-319-70987-1_94

Such fundamentally different are the model based on the resonance-vibrational scheme, and the model based on the shear-shear scheme of structural destruction. The last one is realized when propagating along the construction of a traveling transverse wave.

2 Materials and Methods

Consider the conditions under which the formation of cracks in the path of a wave has a single or multiple character. We assume equation of a plane wave in the form of

$$u(x,t) = A_u cos(\omega t - kx + \varphi) \tag{1}$$

where A_u - amplitude of oscillation; ω - circular frequency; c - wave front speed; φ - initial phase; t – time; - coordinate along which the wave propagates; $k = \omega/c = 2\pi/\lambda$ – wave number (- wavelength). His total energy of the wave (W), equal to the sum of the kinetic (K) and potential (P) energies at each instant of time, is, if the viscous resistance of the medium is neglected, a constant value:

$$\mathrm{W = K + P = m\omega^2 A_u^2/2}, \tag{2}$$

m - mass of oscillating particles.

In the process of particle vibrations, only the conversion of potential energy to kinetic energy occurs and vice versa, while maintaining their total value. The average value of the kinetic energy $(\overline{K})$ равно is equal to the average value of the potential energy (P) and is equal to half the total energy

$$\mathrm{K = P = W/2 = m\omega^2 A_u^2/4}. \tag{3}$$

Dividing the total energy by the volume $V = m/\rho$, where ρ is the density of the medium material, we obtain the wave energy density

$$w = W/V = \left(m\omega^2 A_u^2/2\right)/(m/\rho) = \rho\,\omega^2 A_u^2/2 \tag{4}$$

The quantity equal to the energy transferred by the wave through unit area S per unit time is called the wave intensity (or energy flux density)

$$I = \Delta W/(S\Delta t) = P/S, \tag{5}$$

where $P = \Delta W/(S\Delta t)$ - wave power.

By the time $\Delta t \gg T$ (T – Period of fluctuation) through the surface S will be contained in the volume $\Delta V = S \cdot c \cdot \Delta t$ energy $\Delta W = w\Delta V = w \cdot S \cdot c \cdot \Delta t$, where c – wave speed. Substitution of the obtained ΔW, and also w from (4) into (5) yields:

$$I = \Delta W/(S\Delta t) = wSc\Delta t/(S\Delta t) = wc = c\rho\omega^2 A_u^2/2 \tag{6}$$

$$P = \Delta W/\Delta t = w \cdot S \cdot c \cdot \Delta t/\Delta t = w \cdot S \cdot c = c \cdot \rho \cdot \omega^2 \cdot A_u^2 \cdot S/2 \tag{7}$$

Thus, the intensity of the wave is proportional to the seismic rigidity (c * ρ), the square of the circular frequency and the square of the amplitude. If instead of the circular frequency substitute its expression in a period $\omega = 2\pi/T$, obtain

$$I = 2\pi^2 c \cdot \rho \cdot A_u^2/T^2 \tag{8}$$

When a crack is formed at each point on the line of its development, energy is released $R^2/(2E)$, where R - Resistance of the material to separation, E - modulus of elasticity.

The release of energy due to crack opening occurs not only on its path, but also in the adjacent areas up to l_{crc} в Both sides of the crack, that is, on the square $0.5 \cdot l_{crc} \cdot 2l_{crc} = l_{crc}^2$ or in volume $b \cdot l_{crc}^2$, where b – Section width, l_{crc} – crack length. The intensity of the released energy upon opening the crack is obtained by dividing the energy of the volume by the cross-sectional area and by the time Δt_{crc}, During which the crack passed the path l_{crc}: $\Delta t_{crc} = l_{crc}/c_{crc}$, where c_{crc} - Crack propagation velocity.

Thus, the intensity of the energy released when the crack opens:

$$\begin{aligned} I_{crc} &= \left[R^2/(2E)\right] \cdot b \cdot l_{crc}^2/(bh \cdot \Delta t_{crc}) \\ &= \left[R^2/(2E)\right] \cdot b \cdot l_{crc}^2/(bh \cdot l_{crc}/c_{crc}) = \left[R^2/(2E)\right] \cdot \varsigma \cdot c_{crc}, \end{aligned} \tag{9}$$

here $\varsigma = l_{crc}/h$ – the relative length of the crack in comparison with the size of the sectional height, in the direction of which the crack extends.

3 Results and Discussion

We make approximate estimates of the quantities determined by formulas (8) and (9).

Let the seismic rigidity of the building be 1.05 * 10^6 kg * m^{-2} * s^{-1} (Taken from the book [1] with an effective (averaged) density ρ_e = 700 kg/m^3 and effective speed of propagation of shear waves c_{es} = 1500 m/s, the oscillation period will beand an amplitude of 0.1 m. The intensity of the seismic wave according to the formula (8) will be equal to:

$$I = \frac{2\pi^2}{T^2} \cdot c\rho \cdot A_u^2 = 2 \cdot \frac{3.14^2}{0.4^2} \cdot 1.05 \cdot 10^6 \cdot 0.1^2 = 1.295 \cdot 10^6 \mathrm{N \cdot m}/\left(\mathrm{m}^2 \cdot \mathrm{s}\right)$$

For calculations using formula (9), we take a material (for example, concrete) with the following characteristics:

$$\mathrm{R} = 2\,\mathrm{MPa} = 2 * 10^6 \mathrm{N/m^2}, \mathrm{E} = 20000\,\mathrm{MPa} = 2 * 10^{10} \mathrm{N/m^2}, \mathrm{c_{crc}} = 200\,\mathrm{m/s}, \varsigma = 0.6.$$

With these data, the intensity of energy release during the propagation of a crack will be:

$$I_{crc} = [R^2/(2E)] \cdot \varsigma \cdot c_{crc} = \left[\left(2 \cdot 10^6\right)^2/\left(2 \cdot 2 \cdot 10^{10}\right)\right] \cdot 0.6 \cdot 200 = 0.012 \cdot 10^6$$

As it is seen, the velocity of wave propagation also significantly exceeds the speed of propagation of a crack (c_{es} = 1500 m/s > c_{crc} = 200 m/s). Under these conditions, the development of cracks will be of a multiple nature, since the energy of the wave is too great to form a single crack and, due to a much higher velocity, the wave "runs away" from it, carrying its energy to form new and new cracks. In order to form no more than one crack, the following condition must be fulfilled:

$$I/I_{crc} = (2\pi^2/T^2 \cdot c\rho \cdot A_u^2)/\{\left[R^2/(2E)\right] \cdot \varsigma \cdot c_{crc}\} < 2, \tag{10}$$

From which the relation

$$c/c_{crc} < 2 \cdot R^2 \varsigma \cdot T^2/[(2\pi^2 \cdot 2E) \cdot \rho \cdot A_u^2]. \tag{11}$$

We substitute in (11) the data values used above:

$$c/c_{crc} < 2 \cdot \left(2 \cdot 10^6\right)^2 \cdot 0.6 \cdot 0.4^2 \cdot /(2 \cdot 3.14^2 \cdot 2 \cdot 2 \cdot 10^{10} \cdot 700 \cdot 0.1^2) = 0.139.$$

The value obtained has no physical meaning, since the wave velocity cannot be less than the crack speed. This is because the speed of the wave depends on the modulus of elasticity, density, and, in general, on the coefficient of transverse deformation. Its dependence on the structure of the material is mediated, since both the modulus of elasticity and density are related to the structure. The speed of crack propagation directly depends on the structure: the larger the total contact surface between the heterogeneous components of the material, the smaller the average (or effective) crack propagation velocity, since the cracks propagate predominantly over the contact surfaces between the matrix and the filler. This happens often (though not always) even when the strength of the filler is lower than the strength of the matrix, since the boundaries of dissimilar materials localize the features of the stress and strain fields. In a perfectly homogeneous material, the propagation velocity of the wave and the propagation speed of the crack coincide. In an inhomogeneous material, the propagation velocity of the crack will be less than the velocity of propagation of the wave, and the smaller, the larger the inhomogeneity index.

Therefore, only the multiple formation of cracks corresponds to the chosen values of the parameters. But if the amplitude is half that of the oscillation period, and the period of the oscillations is one and a half times higher than the above, the numerator of the ratio (11) will increase by 2.25 times, the denominator will decrease by 4 times, and the ratio (11) will be greater than unity, which is physically possible and corresponds to the case Formation of a single crack.

We perform simple transformations for the ratio of the intensity of the wave (8) and the intensity of energy release when the crack is opened (9):

$$I/I_{crc} = (2\pi^2/T^2 \cdot c\rho \cdot A_u^2)/\{[R^2/(2E)] \cdot \varsigma \cdot c_{crc}\} = 4\pi \cdot c\rho \cdot A_u^2 \cdot E/(T^2 \cdot R^2 \cdot \varsigma \cdot c_{crc}).$$

Bearing in mind the expression of the wave velocity through the modulus of elasticity and density $c = \sqrt{(E/\rho)}$, wherefrom $E = c^2\rho$, expression of the amplitude of the wave period A_u and the acceleration of gravity (g):

$T = 2\pi(A_u/g)^{1/2}$, - where from $T^2 = 4\pi^2 A_u/g$, and also by introducing designation $k_c = c/c_{crc}$, we obtain:

$$I/I_{crc} = 4\pi^2 \cdot (k_c/\varsigma) \cdot (c\rho)^2 \cdot A_u^2/(4\pi^2 A_u/g \cdot R^2) = (k_c/\varsigma) \cdot (c\rho)^2 \cdot A_u \cdot g/R^2. \quad (12)$$

If in the process of wave propagation no cracks are formed, inequality

$$(k_c/\varsigma) \cdot (c\rho)^2 \cdot A_u \cdot g/R^2 < 1 \quad (13)$$

Or the inequality resulting from it

$$A_u < \varsigma/(k_c g) \cdot [R/(c\rho)]^2 \quad (14)$$

In the last expression, we can get rid of the speed of the wave (c), using the substitution $c = \sqrt{(E/\rho)}$. Then (14) takes the form

$$A_u < \varsigma/(k_c g) \cdot [R/(c\rho)]^2 = \varsigma/(k_c g) \cdot R^2/(E \cdot \rho), \quad (15)$$

More convenient in the sense that instead of the speed of the wave, the value of the elasticity modulus more often encountered and experimentally simpler is used.

With the relative depth of the crack in the reinforced concrete structure (in comparison with the height of the cross section) $\varsigma = 0.6$, acceleration of gravity g = 10 m/s^2, tensile strength of concrete R = 2 * 10^6 Pa, elastic modulus 25 * 10^9 Pa and the density of concrete 2400 kg/m^3 the relation between the amplitude of the wave and the parameter k_c, which is the ratio of the velocity of propagation of the wave to the propagation velocity of the crack, appears in the form

$$A_u < (\varsigma/g) \cdot [R^2/(E \cdot \rho)]/k_c = (0,6/10) \cdot [(2 \cdot 10^6)^2/(2 \cdot 10^9 \cdot 2400)]/k_c = 0.004/k_c$$

The restriction imposed on the amplitude of the wave to avoid the appearance of cracks is proportional to the square of the tensile strength of the material and inversely proportional to the square of the seismic rigidity $(E \cdot \rho = (c \cdot \rho)\,2)$ and inversely proportional to the ratio of the propagation velocities of the wave and crack k_c.

The joint action of tangential stresses on shear forces and normal stresses caused by longitudinal and flexural forces leads to the formation of inclined cracks. It should be noted a very important feature of the development of inclined cracks, due to the presence of two mechanisms of destruction - detachment (o) and shear (c).

Flexural normal cracks near the compressed zone, the size of which is determined by the reinforcement factor, are braked, the detachment mechanism ceases to function (the stretching region before the tip of the crack contracts to the point), and the concrete of the compressed zone is destroyed by the crushing mechanism, crushing. The presence of cracks in the stretched zone has no effect on the course of failure in the final stage, the leveling "plastic" deformations make the stress diagram in the compressed zone close to rectangular, as a result of which the calculated and experimental destructive forces are well coordinated without considering the process of crack development.

A different picture takes place when an inclined crack develops. In the final (or "terminal") stage, the destruction of concrete in front of a crack occurs as a result of a "cut", that is, of the type (c), while the region of the stress and strain field before the tip of the crack does not contract to a point, as in the case of peeled cracks. As a result, disregard for the efforts that are being made in this area can lead to significant errors. That is, with the destruction of the element from the action of transverse force, the entire process, beginning from the stage of cracking and ending with the exhaustion of the bearing capacity, proceeds as a process of development of a crack.

The account of shear deformations under transverse action is accompanied by a decrease in the propagation velocity of forces, which is close to the velocity of transverse waves. Without this account, this velocity is close to the velocity of longitudinal waves. The wave nature of the seismic action reflected in the calculation model causes a slight change in the bending moments in comparison with the results of the calculation without taking into account the wave effects, but shows a significant increase in the transverse forces. This is an important circumstance that prompts a serious adjustment of the resonance vibrational computational model.

Seismic action creates transverse forces of large magnitude in the vertical bearing structures of structures. Tangent and compressive normal stresses along horizontal platforms cause the appearance of the main tensile stresses along the areas oriented at an angle to the initial ones:

$$\alpha = 0.5 \cdot arctg(2\tau/\sigma_0).$$

In the case when the tangential stresses τ caused by the seismic load significantly exceed the normal stress σ_0 from the intrinsic weight, the angle α is close to $\pi/4$.

Main tensile stresses $\sigma = \sqrt{(\sigma_0^2/4 + \tau^2)} - \sigma_0/2$ lead to the formation of inclined crack, the conditions of which are substantially different from those of dipping fractures in such constructions fairly well studied as a beam under static loading.

The criterion for the development of inclined cracks in beams has the form [2, 8, 9]:

$$K_I^2 + K_{II}^2 = K_{Ic}^2 + K_{IIc}^2,$$

where K_I, K_{Ic} - the stress intensity factor at the tip of the crack, growing under the influence of the detachment mechanism, and its critical value;

K_{II}, K_{IIc} - The stress intensity factor at the tip of a crack that grows under the influence of a shear-shear mechanism, and its critical value.

In vertical structural elements under the action of a seismic load, due to a short but significant excess of K_{II} over K_I, the first term in the above criterion can be neglected; In addition, the direction of the development of the crack is more specific in comparison with the beam, instead of the curvilinear trajectory of cracks in the beams, we obtain a nearly rectilinear along the lines In vertical structural elements under the action of a seismic load, due to a short but significant excess of K_{II} over K_I, the first term in the above criterion can be neglected; In addition, the direction of the development of the crack is more specific in comparison with the beam, instead of the curvilinear trajectory of cracks in the beams, we obtain a nearly rectilinear along the lines $\alpha + \pi/2$.

The dynamic nature of the load imposes additional restrictions on the process of development of cracks. The short duration of the pulse and a significant decrease in the propagation velocity of cracks in the concrete as compared to the speed of sound propagation due to lengthening of the path when traversing large aggregate particles can lead to the fact that the crack does not have time to dissect the cross section during the time of the pulse. In addition, the impulse effect generates multiple micro-fractures that grow simultaneously and mutually hamper development. For the critical density of microfractures, apparently, one can take the size of the plastic region in front of the crack tip in the Irvine model [10].

4 Conclusions

In this concept, many problems have been little investigated, in particular, the speed of crack propagation in materials, the critical values of stress intensity coefficients at the tops of cracks developing by shear and mixed mechanisms, and a number of others. However, the study of the consequences of strong earthquakes leads us to conclude that a new view is needed on the causes of seismic destruction, which could improve the theory of seismic resistance and improve the reliability of building calculations for seismic loading. At the same time, the transverse-shift concept should not be considered as an alternative to the resonant-vibrational concept. Each of them has its own fields of application, it is important to determine these area.

References

1. Krivev, V.: Wave Processes in Structures of Buildings Under Seismic Influences. Nauka, Moscow (1987)
2. Peresypkin, E.: Calculation of Core Reinforced Concrete Elements. Stroiizdat, Moscow (1988)
3. Peresypkin, E., Peresypkin, S.: Criterion for the development of cracks in the transverse shear in reinforced concrete structures under seismic actions. In: Abstracts of the reports of the 1st National Conference on Earthquake-Resistant Construction and Seismic Zoning with International Participation. Poltex, Moscow (2001)
4. Peresypkin, E.: Critical stress intensity factor K_{IIc} of heavy concrete on a fine aggregate. Trud-KubGTU XII, Scientific Journal, Krasnodar, Publishing House of Kuban State Technical University, pp. 167–174 (2002)
5. Yazyev, B.: Vestnik MGSU, Proceedings of Moscow State University of Civil Engineering, pp. 56–61 (2013)
6. Yazyev, B., Chepurnenko, A.: Nauchnoe obozrenie 202–204 (2012)

7. Grinfeldi, G.I., Gorshkov, A.S., Vatin, N.I.: Adv. Mater. Res. **941–944**, 786–799 (2014). https://doi.org/10.4028/www.scientific.net/AMR.941-944.786
8. Litvinov, S.: Flat deformation of inhomogeneous multilayer cylinders with allowance for nonlinear creep. Vestnik MGSU **1**, 128–132 (2010)
9. Potekhin, I.: On modeling of an equally stressed cylinder on the basis of the energy theory of strength. Collection of works, KGSHA, Kostroma, vol. 2, pp. 72–74 (2009)
10. Potekhin, I.: The method of optimizing designs on the basis of solving inverse problems in the theory of elasticity of inhomogeneous bodies. DrSc thesis, Moscow (2009)