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Determination of Coefficients for Defining the Deformations when Calculating Reinforced Concrete Structures on Deformation and Cracking Disclosure

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Abstract. The problem of determining the coefficient of averaging the deformations in tensioned reinforcement when calculating reinforced concrete structures according to the limiting states of the second group is considered. An assessment is made of the effect on it of the adhesion stresses distribution type between the reinforcement and the concrete.

Introduction

When designing reinforced structures, the formation and opening of cracks is as important as the issues of strength. This is due to the use of more and more high-strength concrete and reinforcing steel, in which the increase in strength properties is ahead of the improvement of deformative characteristics. As a result, it often happens that the design requirements for strength are met when the requirements for deflection and crack opening are not satisfied.

After the formation of cracks, the stress-strain state of the structure changes abruptly: the deformations in the tensioned reinforcement and in the compressed concrete become uneven: in the section with the crack they get the highest values, in the middle between the cracks - the smallest. Professor V.I. Murashev, the first to develop a method for calculating cracks and deformations of reinforced concrete structures with cracks, introduced the coefficients of averaging of deformations in concrete (ψ_b) and reinforcement (ψ_s) [1] over the length of the section between cracks. This allows us to consider a structure with cracks as if there are no cracks in it, but the presence of cracks causes a change in the deformation characteristics of materials, expressed by averaging coefficients. The coefficient ψ_b has a relatively small range of variability according to numerous experimental data. The coefficient of averaging of deformations in reinforcement ψ_s due to dependence on several factors, and primarily on the nature of distribution along the length of the element and the level of stresses in the reinforcement, varies widely, and therefore its role in determining the curvatures, deflections, distances between cracks, the width of their disclosure is very large.

Research Objective

In the process of changing the stress-strain state of the structure, the distribution of adhesion stresses between the reinforcement and the concrete undergoes significant changes due to the formation of normal cracks in the concrete of the stretched zone, the formation of longitudinal cracks along the reinforcing bars that break the reinforcement adhesion to the concrete in the region of the normal crack at a high level of stresses, and possible other reasons. The variability of the averaging coefficient ψ_b in the process of changing the stress-strain state due to the remoteness of the compressed face from the stretched and self-equilibrium adhesion stresses between the reinforcement and the concrete will be neglected. The purpose of this work is to determine the effect of changes in the adhesion forces between reinforcement and concrete on the value of the averaging coefficient ψ_s.

Material and Research Methods

The subject of study is the coefficient of deformations averaging in tensile reinforcement in bending and eccentric-loaded reinforced concrete elements after the cracks formation. The research method is analytical.

We will proceed from the design scheme of a bent reinforced concrete element based on a nonlinear deformation model [2], [3], adopted in modern norms [4]. This design diagram, in which the diagrams of the state of concrete and reinforcement are taken according to the type of Prandtl diagram and the hypothesis of flat sections is used, is shown in Figure 1.

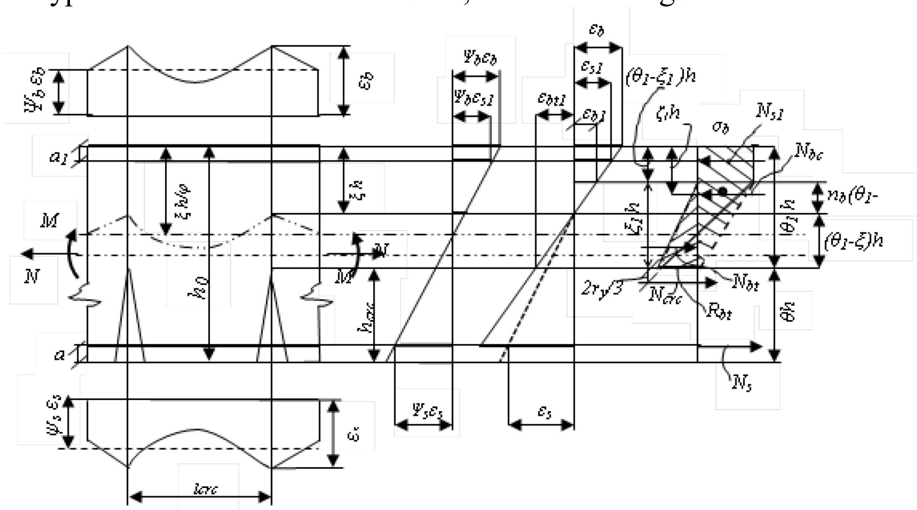

Fig. 1. Design diagram of the reinforced concrete element with cracks during non-linear deformation of the compressed concrete zone

Results of Research and their Discussion

When a crack is formed, its banks are shifted relative to the reinforcement in the direction from the section with the crack. The effect of reinforcement on displaced concrete is directed oppositely to the crack. In this case, the greatest stresses in adhesion occur where shear deformations are maximal. At the initial stage of the formation of normal cracks, such deformations occur directly at the crack. As the level of stresses increases in the site of the greatest shear deformations, the formation of longitudinal cracks that break the reinforcement adhesion to the concrete and the maximum of the adhesion stresses to the middle distance between adjacent normal cracks is possible.

Figure 2 (a) shows some theoretically possible distribution of adhesion stresses between the reinforcement and concrete in the element being bent after the cracks formation, and in Figure 2 b) the forces acting on an elementary small portion of the reinforcement bar with length dx.

From the scheme in figure 2 b) the balance equation has the view:

$$N(x)+2\pi rn\tau(x)dx=N(x)+dN(x),$$

where $2\pi rn\tau(x)dx$ is an effort from bond stresses $\tau(x)$ between reinforcement and the concrete, operating on perimeter n reinforcing bars in the stretched working area at length dx.

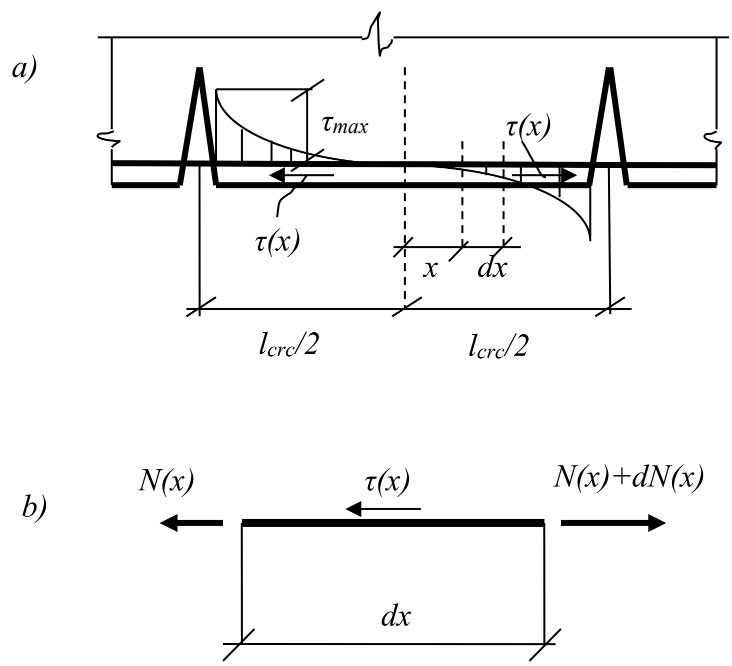

Fig. 2. Scheme for the forces determination in the reinforcement along the length of the section between the cracks: a) - distribution of adhesion stresses between the tension reinforcement and concrete; b) - forces acting on an elementary small portion of the length of the rebar

The equilibrium equation follows from the diagram in Figure 2 b):

$$dN(x)=2\pi rn\tau(x). \qquad (1)$$

The integration of the expression (1) gives

$$N(x)=\int 2\pi rn\,\tau(x)dx + C = 2\pi rnJ_1(x) + C,$$

where $J_1(x)=\int \tau(x)dx$, and C is the constant of integration found from the decision of a problem, which settlement scheme is presented in Figure 1:

$$C=N_s-2\pi rnJ_1(a);$$

N_s – is a stress in reinforcement with a crack, $a=l_{crc}/2$ - half of crack spacing ($Ns=N(a)$).

Taking into account the value of the integration constant "C", the force in the reinforcement in an arbitrary section "x" will take the form:

$$N(x)=2\pi rn\,[J_1(x)-J_1(a)]+N_s. \qquad (2)$$

Find the average value of the force in the N_{sm} reinforcement within the area between the cracks:

$$N_{sm}=\frac{1}{a}\int_0^a N(x)dx = \frac{2\pi rn}{a}[J_2(a)-J_2(0)]-2\pi rnJ_1(a)+N_s, \qquad (3)$$

where $J_2(x)=\iint\tau(x)dxdx$.

The coefficient of the strains averaging in reinforcement is defined as the ratio of the average strain to the maximum. Since at the operational stage deformations in the reinforcement should not reach the yield strength, which means a transition to the limiting state, the value of the coefficient ψ_s can be defined as the ratio of the average force in the reinforcement to the force in section with a crack, which is maximum:

$$\psi_s=N_{sm}/N_s=2\pi rn[J_2(a)-J_2(0)]/(aN_s)-2\pi rnJ_1(a)/N_s+1. \quad (4)$$

We introduce the notation $\gamma 0=lcrc/h0$, where $h0$ – is a working height of the section, that is, the height of the cross section minus the thickness of the protective layer of the working reinforcement. Then

$$l_{crc}=\gamma_0 h_0, \quad a=lcrc/2=\gamma\ 0h0/2. \quad (5)$$

The force in the reinforcement in the cross section with a crack is equal to the product of the cross-sectional area of the reinforcement $\pi r^2 n$ and the corresponding stress in it σ_s:

$$N_s=\pi r^2 n\sigma_s. \quad (6)$$

Having substituted (5) and (6) in (4) after simple transformations we get:

$$\psi_s=\frac{4}{\gamma_0 h_0 r\sigma_s}[J_2(a)-J_2(0)]-\frac{2}{r\sigma_s}J_1(a)+1. \quad (7)$$

The obtained formula (7) differs from the corresponding expression (13) in the book [5]. This difference is due to the fact that in the book [5] the area of reinforcement is considered to be evenly “spread” across the width of the section of element b, as a result of which the adhesion stresses to concrete are also distributed across the width b. Here, reinforcement is considered in the form of n individual rods of radius r and, therefore, adhesion stresses $\tau\ (x)$ act along the perimeter of all rods $p = 2\pi rn$. That is, in this case, the total perimeter of the reinforcing bars corresponds to the width of the section:

$$b=2\pi rn. \quad (8)$$

Rebar cross-sectional area is $A_s=\pi r^2 n$. On the other hand, $A_s=\mu bh_0$, where μ is a reinforcement ratio equal to the ratio of A_s reinforcement cross-sectional area to bh_0 concrete cross-sectional area. Taking into account (8), $A_s=\mu bh_0=2\pi rn\mu h_0$.

Thus, $A_s=\pi r^2 n$ and $A_s=2\pi rn\mu h_0$. Equating the right-hand side of different expressions of the cross-sectional area of the reinforcement, we get:

$$r=2\mu h_0. \quad (9)$$

The substitution (9) in (7) leads to an expression that completely coincides with expression (13) in [5]. The formula (7) obtained here has the advantage of explicitly taking into account the effect of the size of the reinforcement cross-section on the deformations averaging coefficient ψ_s.

Considering several different functions of the distribution of adhesion stresses along the length of the element between adjacent cracks:

$$\tau_1(x)=Tx/a, \quad (10)$$
$$\tau_2(x)=Tsin(\pi x/(2a)), \quad (11)$$
$$\tau_3(x)=Tsin(\pi x/a), \quad (12)$$

Where T is the peak value of bond stresses set according to size standards [4].

Formulas (10) and (11) correspond to the cases when the maximum adhesion stresses occur near the cracks, where due to crack opening, the greatest shifts of concrete relative to the reinforcement occur. Formula (12) displays those features of concrete deformation near the cracks formed, when the concrete shifts relative to reinforcement are so great that cracks of separation of concrete from reinforcement are formed, breaking the adhesion between reinforcement and concrete near the crack coast. In this case, the maximum of the clutch plots is shifted from the crack in the direction of the middle distance between the cracks.

Let's calculate the values of integrals $J_1(a)$, $J_2(a)$, $J_2(0)$ in the formula (7). For function $\tau_1(x)$ (10) these integrals will be designated $J_{1,1}(a)$, $J_{2,1}(a)$, $J_{2,1}(0)$, for function $\tau_2(x)$ (11) – according to $J_{1,2}(a)$, $J_{2,2}(a)$, $J_{2,2}(0)$ and for the function $\tau_3(x)$ (12) - $J_{1,3}(a)$, $J_{2,3}(a)$, $J_{2,3}(0)$.

$$J_{1,1}(x)=\int\tau_1(x)dx=\int(Tx/a)dx=Tx^2/(2a),\;\; J_{1,1}(a)=Ta/2;$$

$$J_{2,1}(x)=\iint\tau_1(x)dxdx=Tx^3/(6a),\;\; J_{2,1}(a)=Ta^2/6,\;\; J_{2,1}(0)=0;$$

$$\psi_{s1}=\frac{4}{\gamma_0 h_0 r\sigma_s}[J_{2,1}(a)-J_{2,1}(0)]-\frac{2}{r\sigma_s}J_{1,1}(a)+1=\frac{4}{2ar\sigma_s}Ta^2/6-\frac{2}{r\sigma_s}Ta/2+1=$$
$$=\frac{Ta}{3r\sigma_s}-\frac{Ta}{r\sigma_s}+1=1-\frac{2}{3}\frac{Ta}{r\sigma_s} \tag{13}$$

$$J_{1,2}(x)=\int\tau_2(x)dx=\int(T\sin\frac{\pi x}{2a})dx=-\frac{2aT}{\pi}\cos\frac{\pi x}{2a},\;\; J_{1,2}(a)=0;$$

$$J_{2,2}(x)=\iint\tau_2(x)dxdx=-\frac{4a^2T}{\pi^2}\sin\frac{\pi x}{2a},\; J_{2,2}(a)=-\frac{4a^2T}{\pi^2},\;\; J_{2,2}(0)=0;$$

$$\psi_{s2}=\frac{4}{\gamma_0 h_0 r\sigma_s}[J_{2,2}(a)-J_{2,2}(0)]-\frac{2}{r\sigma_s}J_{1,2}(a)+1=-\frac{4}{2ar\sigma_s}4Ta^2/\pi^2-\frac{2}{r\sigma_s}*0+1=$$
$$=-\frac{8Ta}{\pi^2 r\sigma_s}+1=1-\frac{8}{\pi^2}\frac{Ta}{r\sigma_s}. \tag{14}$$

Similarly, the averaging coefficient ψ_{s3} is obtained for the coupling stress distribution function in the form (12):

$$\psi_{s2}=1-\frac{2}{\pi}\frac{Ta}{r\sigma_s}. \tag{15}$$

Noticing in (13) - (15) that the structure of all formulas is the same, and the second terms differ only by one factor, we write these formulas in the generalized form:

$$\psi_{s,i}=1-k_{f,i}Ta/(r\sigma_s),\;\; i=1,\,2,\,3 \tag{16}$$

where $k_{f,i}$ – is an Adhesion Shape Coefficient:

$$k_{f,i}=\begin{cases}2/3, & \tau_1(x)=Tx/a,\\ 8/\pi^2, & \tau_2(x)=T\sin(\pi x/(2a)),\\ 2/\pi, & \tau_3(x)=T\sin(\pi x/a)\end{cases}$$

Figure 3 shows the dependences of the averaging coefficient on the level of stresses in the reinforcement for different forms of adhesion diagrams by the relations (16).

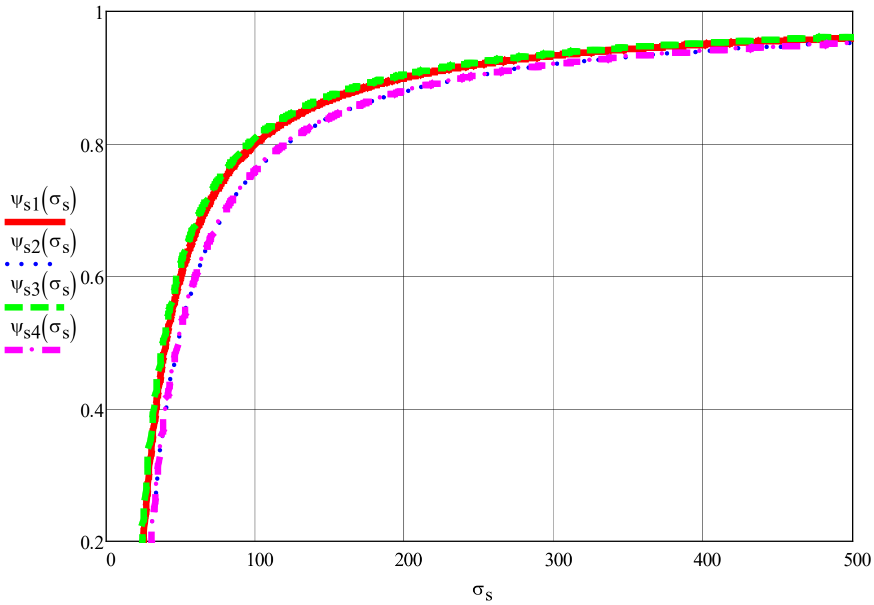

Fig. 3. Dependence of the averaging coefficient ψs on the shape of the coupling diagram and the stress level in the reinforcement *σs (T = 3 MPa, a / r = 10*; ψ_{s4} - according to the norms [4])

As it can be seen from the Figure 3, even significantly differing in shape of the armature adhesion coupling with concrete, represented by a linear function (10) with a maximum at the crack and a sinusoid (12) with a maximum displaced from the crack, give almost coincident graphs. For comparison, a graph of the dependence of the averaging coefficient ψ_{s4} on the stresses in the reinforcement, constructed according to the formula (8.137) of the norms [4]$\psi_s=1-0,8\sigma_{s,crc}/\sigma_s$ where $\sigma_{s,crc}$ -is a stress in tensioned reinforcement in the formation of cracks, taken here equal to 30 MPa, which corresponds to the limit tensile concrete $\varepsilon_{bt}=0,15*10^{-3}$. The curve according to the norm formula almost completely coincides with the ψ_{s2} curve, and the difference between all the examined dependences of the averaging factor on the stresses in the reinforcement does not exceed 5% anywhere.

Summary

The analysis performed allows us to conclude that the shape of the articulation bond between reinforcement and concrete has little effect on the magnitude of the strain averaging coefficients in the tensile reinforcement of the reinforced concrete element. In practical calculations, it is possible to use the simplest forms, for example, linear, without a large error.

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